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# A Closer Look at Perturbative Corrections in the $b \rightarrow c$ Semileptonic Transitions

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## Abstract

We comment on two recent calculations of the second order perturbative corrections in the heavy flavor semileptonic transitions within the Brodsky-Lepage-Mackenzie approach. It is pointed out that the results do not show significant enhancement either in the inclusive  $b \rightarrow c$  decays or in the exclusive amplitudes at zero recoil provided that the expansion parameter is chosen in a way appropriate to the kinematics at hand. The values of the second-order coefficients inferred from the BLM-type calculations appear to be of order unity in the both cases. Thus, in both cases no significant uncertainty in extracting  $V_{cb}$  can be attributed to perturbative effects. The theoretical accuracy is mostly determined by the existing uncertainty in  $1/m_c^2$  nonperturbative corrections in the exclusive  $B \rightarrow D^*$  amplitude, and, to a lesser extent, by the uncertainty in the estimated value of the kinetic-energy matrix element  $\mu_\pi^2$  in the case of  $\Gamma_{sl}(b \rightarrow c)$ . The theoretical accuracy of the inclusive method of determination of  $V_{cb}$  seemingly competes with and even exceeds the experimental accuracy.

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In the last two years considerable progress has been achieved in the theory of nonperturbative effects in the heavy flavor decays. This theory has been applied to many problems from  $B$  physics, including such practically important application as precise determination of  $V_{cb}$  from experimental data. Further work in this direction is under way.

The current level of understanding of the nonperturbative effects is such that, surprisingly, calculation of the ‘trivial’ perturbative corrections due to hard gluon exchanges becomes the bottle neck of theoretical analysis. The perturbative corrections are often suspected to limit the theoretical accuracy. In most of the processes of interest only one-loop corrections are fully calculated at present. To match the level of accuracy obtained in the nonperturbative sector one needs to know two-loop  $\mathcal{O}(\alpha_s^2)$  terms.

Complete calculation of these terms typically is a rather difficult task. Sooner or later it has to be addressed since without the complete two-loop results no analysis of the heavy flavor decays can be considered as fully conclusive. At present such calculations are not available, and one has to settle for less.

It is known for a long time that  $b$ , the first coefficient in the Gell-Mann-Low function, is a large numerical parameter in QCD. In many cases, when both  $\mathcal{O}(\alpha_s)$  and  $\mathcal{O}(\alpha_s^2)$  corrections are known, the set of  $(\alpha_s/\pi)^2$  terms is dominated by those graphs that are related to the running of  $\alpha_s$ .<sup>1</sup> The sum of all these graphs is proportional to  $b(\alpha_s/\pi)^2$ , while all other two-loop graphs yield  $(\alpha_s/\pi)^2$  with a coefficient of order one. This observation gives rise to the so called BLM hypothesis [2] according to which a good idea of the  $(\alpha_s/\pi)^2$  terms can be obtained by calculating this “running  $\alpha_s$ ” subset of the two-loop graphs while ignoring all other diagrams.

In two recent stimulating papers the BLM approach has been used for estimating the  $\alpha_s^2$  corrections in the  $b \rightarrow c$  transition. In Ref. [3] (initiated by works [1, 4]) this correction was considered in the exclusive  $b \rightarrow c$  amplitude at zero recoil and was found to be small. (By small we mean that the coefficient in front of  $(\alpha_s/\pi)^2$  is of order unity.) A similar calculation was carried out in Ref. [5] for the inclusive width of the  $b \rightarrow c$  semileptonic decay with the conclusion that the  $\alpha_s^2$  terms are large. This result was interpreted (see [3]) as a demonstration of an uncontrollable character of the perturbative series in the inclusive decays which allegedly blocks [6] accurate determination of  $V_{cb}$  within the inclusive method.

These two assertions above – small two-loop corrections in the exclusive case and large in the inclusive one – seemingly clash with each other if one combines them with the facts that (i) in the small velocity (SV) limit [7] the total inclusive probability is completely saturated by the exclusive transitions  $B \rightarrow D^*$  and  $B \rightarrow D$ ; (ii) in the  $b \rightarrow c$  semileptonic decays we are actually rather close to the SV limit – in the essential part of the phase space the velocity of the  $D$  ( $D^*$ ) is small in the  $B$  rest frame. The proximity to the SV limit is substantiated by experimental data [8] according to which only 25 to 30% of the total probability leaks into non-

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<sup>1</sup>Let us parenthetically note that in the *Coulomb gauge* these are merely the graphs with the polarization operator insertions in the gluon line [1].

elastic channels (i.e. those other than  $B \rightarrow Dl\nu$  and  $B \rightarrow D^*l\nu$ ). The theoretical clarification of this experimental fact goes beyond the scope of the present paper and is discussed elsewhere.

In this letter we argue that the paradox above is superficial. Analyzing the exclusive and inclusive channels in parallel and expressing the perturbative series in both cases in terms of one and the same coupling constant (the one which is *physically relevant*, see below) we eliminate the apparent qualitative contradiction between the results for inclusive and exclusive transitions and, moreover, demonstrate that the  $\alpha_s^2$  terms are small, so that they cannot be reliably captured within the BLM approach *per se*. The present consideration, thus, gives further support to the observation [9] that the most accurate way of extracting  $V_{cb}$  available now is the analysis of the inclusive semileptonic rate where the theory of the nonperturbative effects is much more predictive than in the exclusive decays.

To elucidate our point we need to do some preparatory work. Let us start from the corrections in the exclusive  $b \rightarrow c$  amplitudes at zero recoil. The one loop result for the axial form factor  $\eta_A$  at zero recoil is well known [7],

$$\eta_A = 1 + \frac{\alpha_s}{\pi} \left( \frac{m_b + m_c}{m_b - m_c} \ln \frac{m_b}{m_c} - \frac{8}{3} \right) ; \quad (1)$$

The analysis of Ref. [10] establishes that the normalization point of the gauge coupling  $\alpha_s$  that is physically adequate in this case is

$$m_0 = \sqrt{m_c m_b} . \quad (2)$$

In other words, if one expresses the perturbative series for  $\eta_{V,A}$  in terms of  $\alpha_s(m_0)$  no large numbers appear in the expansion coefficients at order  $\alpha_s^2$ . (Below, if not indicated explicitly,  $\alpha_s$  means, by definition,  $\alpha_s(m_0)$ .) This leads to a numerical estimate <sup>2</sup>  $\eta_A \simeq 0.965$ .

At the two-loop level it was found in Ref. [3] that in the  $\overline{\text{MS}}$  scheme

$$\eta_A = 1 - 0.431 \frac{\overline{\alpha}_s(m_0)}{\pi} - 1.211 \left( \frac{\overline{\alpha}_s}{\pi} \right)^2 . \quad (3)$$

The values of the coefficients above refer to the ratio of the quark masses

$$z \equiv m_c/m_b = 0.3 . \quad (4)$$

We will adopt this value of  $z$  throughout the paper for the sake of definiteness. All quantities referring to the  $\overline{\text{MS}}$  scheme are marked by bars. The following generic notation will be used:

$$\eta_A = 1 + a_1 \frac{\alpha_s}{\pi} + a_2 \left( \frac{\alpha_s}{\pi} \right)^2 + \dots \quad (5)$$

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<sup>2</sup>Some partial higher order calculations of the  $b \rightarrow c$  zero recoil amplitudes which existed in the literature previously [11, 12, 6] and gave rise to the estimate  $\eta_A = 0.986 \pm 0.006$  were shown [10] to be irrelevant.

It is quite evident that the perturbative coefficients starting from the second order depend on the scheme used to define the strong coupling. The number for the two-loop coefficient in Eq. (3) is given in the  $\overline{\text{MS}}$  scheme often used in calculations performed in dimensional regularization. This scheme uses certain rather *ad hoc* subtraction constants which are introduced for technical purposes only, and is quite unphysical. Moreover, such a choice is definitely unnatural for the calculations where one accounts only for the running of  $\alpha_s$ . The so-called  $V$  scheme of BLM [2] is physically preferable here reflecting the real size of the effect. As a matter of fact, the actual calculations within the BLM approach are routinely performed just in this scheme (for details see [13, 3, 5]), and only at the final stage is the series reexpressed in terms of  $\alpha_s$  in the  $\overline{\text{MS}}$  scheme and  $\bar{a}_2$  introduced. It is more adequate to use the  $V$ -scheme strong coupling, at least at the current stage when the calculations beyond the BLM level have not been done. In what follows we will work with the  $V$ -scheme  $\alpha_s$ , unless otherwise indicated;  $\alpha_s$  no bar refers to the  $V$  scheme while  $\bar{\alpha}_s$  is reserved for the  $\overline{\text{MS}}$  coupling.

The one loop relation between  $\bar{\alpha}_s$  and  $\alpha_s$  is given by

$$\frac{\alpha_s(\mu)}{\pi} = \frac{\bar{\alpha}_s(\mu)}{\pi} + \frac{5}{12}b \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \quad (6)$$

and, therefore, the perturbative expansion in the  $\overline{\text{MS}}$  scheme

$$\eta_A = 1 + \bar{a}_1 \frac{\bar{\alpha}_s(\mu)}{\pi} + \bar{a}_2 \left( \frac{\bar{\alpha}_s(\mu)}{\pi} \right)^2 + \dots$$

takes the form

$$\eta_A = 1 + \bar{a}_1 \frac{\alpha_s(\mu)}{\pi} + \left( \bar{a}_2 - \frac{5}{12}b\bar{a}_1 \right) \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 + \dots \quad (7)$$

so that  $a_1 = \bar{a}_1$  and  $a_2 = \bar{a}_2 - \frac{5}{12}b\bar{a}_1$ . The first three terms in the expansion of  $\eta_A$ , Eq. (3), then look as follows for  $m_c/m_b = 0.3$ :

$$\eta_A = 1 - 0.431 \frac{\alpha_s(m_0)}{\pi} + 0.286 \left( \frac{\alpha_s}{\pi} \right)^2 + \dots \quad (8)$$

where  $m_0$  is defined in Eq. (2). We see that the calculated second-order coefficient is in fact noticeably less than 1, i.e. clearly not enhanced. For this reason it cannot be taken even as an estimate of the second order result because other terms beyond the BLM approximation are expected to be of order 1.

Now let us proceed to the inclusive semileptonic widths to bring the corresponding analysis in line with the exclusive case.

Assume for a moment that  $(m_b - m_c)/(m_b + m_c) \ll 1$ ; then the SV limit is obviously parametrically guaranteed, and the inclusive width is directly expressed in terms of  $\eta_A$  and  $\eta_V$ , the zero recoil form factors of the vector and axial currents:

$$\Gamma_{\text{sl}} = \frac{G_F^2(m_b - m_c)^5}{60\pi^3} (\eta_V^2 + 3\eta_A^2) |V_{cb}|^2 \cdot \left[ 1 + \mathcal{O} \left( \left( \frac{m_b - m_c}{m_b + m_c} \right)^2 \right) \right] . \quad (9)$$

This formula is nothing else than the text-book expression for the neutron  $\beta$ -decay width (with the electron mass neglected). Moreover, in the SV limit the deviation of the zero recoil vector form factor  $\eta_V$  from unity is also proportional to the same smallness parameter having the meaning of the square of the typical velocity in the decay, and thus can be neglected as well. Therefore in the limit when  $(m_b - m_c)/(m_b + m_c) \ll 1$  the perturbative expansions for the inclusive width and for the  $B \rightarrow D^* \ell \nu$  zero recoil amplitude squared are directly related (essentially the same).

In the real world  $m_c$  is significantly smaller than  $m_b$ . However, as was mentioned above, in the semileptonic  $b \rightarrow c$  decays one effectively is not far from the SV limit. The experimental evidence in favor of this fact is the approximate saturation of the experimental inclusive probability by the two elastic channels. It was pointed out in [10] that the coefficients in the perturbative expansion of  $\eta_A$  do not deviate drastically from their values at  $m_c = m_b$ ; this observation was confirmed by explicit calculation in the BLM approximation in Ref. [3]. Since near the SV limit the elastic channel saturation is complete (this, of course, refers also to the perturbative corrections), it is advantageous to write the perturbative series for the inclusive width in terms of the same coupling constant as in the exclusive case,  $\alpha_s(m_0)$ . This choice rules out parametrically large two-loop coefficients in  $\eta$ 's; by the same token it will kill such coefficients in the inclusive width.

Let us examine the result for the inclusive width [5] more carefully taking advantage of the proximity to the SV limit. Let us write down the perturbative expansion for the inclusive width in the form

$$\Gamma_{\text{sl}} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 z_0(z) \kappa_\Gamma^2(z) \ ,$$

$$\kappa_\Gamma(z) = 1 + k_1(z) \frac{\alpha_s}{\pi} + k_2(z) \left( \frac{\alpha_s}{\pi} \right)^2 + \dots \quad (10)$$

where  $z_0(z)$  is the well known phase space factor. Then one has

$$\kappa_\Gamma^2(z) = \frac{1}{4} + \frac{3}{4} \eta_A^2(1) + \mathcal{O}\left((1-z)^2\right) \ . \quad (11)$$

Comparing the expansions for  $\eta_A$  and for  $\kappa_\Gamma$  we get

$$k_1(z) = \frac{3}{4} a_1(1) + \mathcal{O}\left((1-z)^2\right) = -\frac{1}{2} + \mathcal{O}\left((1-z)^2\right) \ ,$$

$$k_2(z) = \frac{3}{4} a_2(1) + \frac{3}{32} a_1^2(1) + \mathcal{O}\left((1-z)^2\right) = \frac{3}{4} a_2(1) + \frac{1}{24} + \mathcal{O}\left((1-z)^2\right) \ . \quad (12)$$

It is essential that terms  $\mathcal{O}((1-z))$  are absent. Examination of Eq. (12) shows that there are no reasons to expect too big a value of  $k_2$  even for the actual masses of  $b$  and  $c$ . It is worth noting here that Eq. (11) assumes the use of the “pole” masses for the  $b$  and  $c$  quarks, and the corresponding definition of the perturbative factor  $\kappa_\Gamma^2$  in Eq. (10); otherwise Eq. (9) does not hold. The notion of the pole mass

cannot be consistently defined to all orders in the coupling constant, see Refs. [1, 4]. Perturbatively it can be defined to a given (finite) order, and for our limited technical purposes of discussing the two-loop perturbative corrections it is legitimate to use the corresponding “two-loop pole” masses.

Of course, for Eq. (12) to hold the strong coupling must be defined in the same way in both cases, inclusive and exclusive. Taking into account the experimental proximity to the SV limit in the semileptonic  $b \rightarrow c$  decays, we will express the perturbative expansion for  $\Gamma_{\text{sl}}$  in terms of  $\alpha_s(m_0)$ , which will allow one a direct comparison with the exclusive decays.

Recently, calculation of the  $\alpha_s^2$  effects in  $\Gamma_{\text{sl}}(b \rightarrow c)$  associated with running of  $\alpha_s$  has been done [5] based on a trick suggested in [13]. For  $m_c/m_b = 0.3$  it was obtained that

$$\kappa_\Gamma^2 = 1 - 1.67 \frac{\overline{\alpha}_s(m_b)}{\pi} - 15.1 \left( \frac{\alpha_s}{\pi} \right)^2. \quad (13)$$

According to Eq. (6) and the one-loop evolution of  $\alpha_s(\mu)$  this expansion in terms of  $\alpha_s(m_0)$  (the proper choice for the kinematics at hand) takes the form

$$\kappa_\Gamma^2 = 1 - 1.67 \frac{\alpha_s(m_0)}{\pi} - 5.11 \left( \frac{\alpha_s}{\pi} \right)^2 + \dots \simeq 1 - 0.16 - 0.047 + \dots \quad (14)$$

where we used for numerical illustration the value  $\alpha_s(m_0) = 0.3$  corresponding to the choice  $\overline{\alpha}_s(m_0) = 0.24$  adopted in Ref. [3]. Perturbative corrections, thus, do not show enhancement and, rather, seem to be well under control; no trace of uncontrollable 10% second-order corrections claimed in [6] is seen. We should note once again that the second-order coefficient in Eq. (14) is not very large; it becomes even smaller if one considers  $\kappa_\Gamma$  instead of  $\kappa_\Gamma^2$  which is reasonable for the purpose of comparison with  $\eta_A$  (see below). If so, this result cannot be taken too literally because other second-order corrections that are beyond BLM are expected to be of a similar size.

Now we turn to the question of theoretical uncertainties in determining  $V_{cb}$ . Following the existing tradition to phrase the perturbative corrections to the  $B \rightarrow D^* \ell \nu$  rate in terms of corrections to the amplitude rather than to the rate itself, we also write the perturbative expansion for the square root of the width,  $\kappa_\Gamma = (\Gamma_{\text{sl}}^{\text{pert}}/\Gamma_{\text{sl}}^0)^{1/2}$ ; this form is directly related to extraction of  $|V_{cb}|$ :

$$\begin{aligned} \eta_A &= 1 - 0.431 \frac{\alpha_s(m_0)}{\pi} + 0.286 \left( \frac{\alpha_s}{\pi} \right)^2 + \dots = 1 - 0.041 + 0.0026 + \dots \\ \kappa_\Gamma &= 1 - 0.835 \frac{\alpha_s(m_0)}{\pi} - 2.90 \left( \frac{\alpha_s}{\pi} \right)^2 + \dots = 1 - 0.08 - 0.026 + \dots \end{aligned} \quad (15)$$

It is obvious that no significant theoretical uncertainty can be attributed to the perturbative corrections in both cases. Moreover, the genuine relation between the second-order coefficients can be established only after the complete two-loop calculations are made: the BLM-type calculation does not give a large contribution that would safely dominate the second-order result.

Continuing the analysis of the numerical aspect of the uncertainties it is appropriate to note that the nonperturbative corrections to the inclusive semileptonic widths are calculated [14, 15, 16] and give the correction

$$(\delta\kappa_\Gamma)_{\text{nonpert}} \simeq -2.5\% \quad . \quad (16)$$

This is smaller than the first order perturbative correction but dominates the uncertainty in the  $\alpha_s^2$  terms. A different situation occurs in the zero recoil  $B \rightarrow D^*$  amplitude. The leading nonperturbative corrections  $\mathcal{O}(1/m_c^2)$  are not fully known here (in a model-independent way); a reasonable estimate of these corrections is about  $-10\%$  [9] with the uncertainty of about  $\pm 4\%$ . Thus, they are definitely larger than even the first order radiative corrections and, therefore, the exact value of the second-order perturbative coefficient is not of much practical relevance here since it is not abnormally large.

To complete our discussion of the purely perturbative uncertainties in  $b \rightarrow c$  transitions, let us mention that at present the value of the strong coupling at the scale  $2.5 \text{ GeV}$  is known, apparently, with the accuracy  $\sim 10\%$ . The corresponding uncertainty in  $\eta_A$  is thus  $\sim 0.004$  and is definitely irrelevant in view of large nonperturbative uncertainties. The corresponding uncertainty in  $\kappa_\Gamma$  which enters determination of  $|V_{cb}|$  from the inclusive width constitutes about  $0.007$ ; being rather small and negligible at present, it may contribute a noticeable (through evidently not dominant) part in the overall theoretical uncertainty in the near future. At the moment the lack of the definite knowledge of the exact second-order coefficient also does not seem to limit the theoretical accuracy of this most precise method of determination of  $|V_{cb}|$ .

Two brief remarks concerning the nonperturbative corrections which were mentioned in passing are in order here. The theoretical basis for their calculation is provided by the Wilson operator product expansion (OPE) [17]. In calculating  $\Gamma_{\text{sl}}(b \rightarrow c)$  OPE is used in the Minkowski domain which assumes duality. Deviations from duality (they die off exponentially) are controlled by the behavior (divergence) of the high-order terms in OPE [18]; they can not be found from purely theoretical considerations at present. On the basis of the phenomenological information available it is reasonable to believe that these deviations are negligible in the  $b \rightarrow c$  transition (unlike in  $c \rightarrow s$  decays [19]). They are not included in the uncertainties in the nonperturbative terms which were quoted above. Another aspect of OPE ignored so far in this letter is the need for introducing a boundary point  $\mu$  in any consistent OPE calculation with the subsequent separation of the short- and large-distance contributions. The latter must be subtracted from the coefficient functions calculated perturbatively since they are accounted for in the matrix elements of higher-dimension operators (condensate terms). In the calculations of the perturbative corrections to  $\eta_A$  [3] and, especially, to the inclusive  $c \rightarrow s \ell \nu$  width [20] the sizable part of the second-order coefficient comes from the low-momentum region of integration where perturbative expressions for gluon and quark propagators are not applicable. Therefore the numerical estimates derived from those calculations

have no direct physical meaning: the impact of this region is completely accounted for by nonperturbative contributions and must be excluded from the perturbative coefficients in the proper treatment (see [1, 21]). If it is done, the size of perturbative corrections in  $c \rightarrow s \ell \nu$  decays reduces essentially. In the case of  $\eta_A$  this modification is numerically larger than the whole second-order correction estimated in Ref. [3]:

$$\eta_A(\mu) \simeq \eta_A + \frac{\alpha_s(\mu)}{3\pi} \mu^2 \left( \frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) \quad (17)$$

where  $\mu$  is the infrared cutoff (for more detail see [21]). The question of the consistent simultaneous account for both nonperturbative and perturbative effects [21] becomes important already at the level of the second-order perturbative corrections when the former are governed by the mass of  $c$  quark, due to the existing numerical hierarchy among the corrections in this case. Although the need for such consistent treatment based on the Wilson OPE is acknowledged, practically this aspect is usually ignored in HQET calculations (see, e.g., [12, 22]).

To summarize, we demonstrated that using the proper perturbative expansion parameter makes the second-order perturbative corrections equally small both in the  $b \rightarrow c \ell \nu$  amplitude squared at zero recoil and in the inclusive  $b \rightarrow c \ell \nu$  width. The existing BLM-type calculations do not show any particular enhancement of the second-order effects associated with the running of  $\alpha_s$  if the scale is chosen in a way appropriate for the SV kinematics. For this reason the recent results [3, 5] on the second-order perturbative corrections in the inclusive and exclusive heavy quark transitions can be viewed mainly as the indication that these corrections are not large and the perturbative series are well controlled in both cases. More precise results can be obtained only by complete two-loop computations; they are necessary for the ultimate improvement of the theoretical precision in the most accurate method based on the inclusive semileptonic width. For exclusive transitions this can not improve the accuracy of extracting  $V_{cb}$  because the uncertainty is by far dominated by rather poorly known nonperturbative corrections. Still, the full two-loop calculation seems to be desirable in this case as well: one can use the value of  $V_{cb}$  obtained from the inclusive widths *to measure* the exclusive form factor with sufficiently high accuracy and, thus, to obtain valuable information on the structure of nonperturbative effects in the  $b \rightarrow c$  transitions in the situation when they are significant.

We emphasize that the concrete method of determining  $V_{cb}$  from the inclusive width at present is based on the fact that the expression for the width valid in the SV kinematics works well for the actual quark mass ratio – the fact known theoretically for a long time and now substantiated by experimental studies of the final charm states showing the relevance of the SV limit. Therefore, the consistent analysis of corrections requires using the expansion parameters inherent to the SV kinematics.

The result of recent calculations of potentially dominant second-order corrections [5] shows that the perturbative uncertainty affecting the extraction of  $V_{cb}$  now does not exceed a percent level and is similar to the perturbative uncertainty in the



exclusive zero recoil transition, contrary to rather *ad hoc* claims existing in the literature [6, 3]. This uncertainty is smaller than the impact of the nonperturbative corrections which are reliably calculated for the inclusive  $b \rightarrow c$  width. On the contrary, the nonperturbative corrections seem to be much larger in the exclusive decays and, moreover, they carry a significant uncertainty that can not be eliminated in a model-independent way at present. This limits the accuracy of the exclusive method of extracting  $V_{cb}$  but, also, offers opportunities of learning more about details of the nonperturbative dynamics.

It seems justified to say that with the recent progress in the theoretical treatment of the heavy flavor transitions the theoretical accuracy in determination of  $V_{cb}$  from  $\Gamma_{sl}(B)$  achieved by year 1995 exceeds the existing experimental accuracy.

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